Course : Algebra 3 **Year : 2023/2024**
Chapter 1 : Determinants of matrices **Batna 2 University**
Department of Computer Science

Tutorial series 1

Exercise 0.1 *Let* $A = \begin{pmatrix} 3 & 5 & 6 \\ a & 4 & 0 \\ b & 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 4 & 1 \\ 0 & 2 & 3 \\ 4 & 1 & 0 \end{pmatrix}$, *and* $C = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$.

1. *Give the values of* a, b *under which* A *is a symmetric (or an upper triangular) matrix.*

2. *Under what conditions can we say that* C *is a diagonal matrix.*

3. *Find* B^T.

4. *Compute* $|B|$, *is it possible to determine* B^{-1} *?*

Exercise 0.2 *Let*

$$A = \begin{pmatrix} 5 & 6 & 7 \\ 2 & 3 & 4 \\ 1 & 5 & 2 \end{pmatrix}.$$

Compute $|A|$, *what can we deduce?*

Exercise 0.3 *Let*

$$A = \begin{pmatrix} \alpha & 1 & 2 \\ 0 & 2 & \alpha \\ \alpha - 2 & 1 & 1 \end{pmatrix}.$$

Try to get the value of α *in the case where* A *is singular.*

Exercise 0.4 *Consider*

$$\begin{array}{ll} 2x + y + 3z = 3, & 3x + 2y + 9z = 4, \\ 4x + 5y + 7z = 0, & 8x + z = 2, \\ y + 8z = 2 & 7x + 5y + 4z = 1 \end{array}$$

1. *Rewrite the above systems in the form* $AX = B$.

2. *By using Cramer's rule, determine the solutions of thse systems.*

Exercise 0.5 *Consider*

$$\begin{array}{ll} 5x + y = 5, & x + 2y + 4z = 1, \\ 3x + 4y = 6, & 7x + 5y + 3z = 3, \\ & 9x + 7y + z = 4 \end{array}$$

1. *Rewrite these systems in the form* $AX = B$.

2. *In each case, check that* A *is invertible and give the inverse of* A.

3. *By using matrix inversion method, find the solutions of thse systems.*

Exercise 0.6 *A is $n \times n$ matrix.*

1. *Prove that $B^2 = -B$, where $B = A - I$ and $A^2 = A$.*

2. *Prove that $B^2 = 6B$, where $B = 3(A + I)$ and $A^2 = I$.*

Exercise 0.7 *A and B are of the same order.*

1. *Prove that, in the case where $|A| \neq 0$,*

$$(A^T)^{-1} = (A^{-1})^T.$$

2. *Prove that, in the case where $|A| \neq 0$ and $|B| \neq 0$,*

$$|AB| \neq 0,$$

and

$$(AB)^{-1} = B^{-1}A^{-1}.$$