Course : Algebra 3   **Year : 2023/2024**
Chapter 2 : Linear systems   **Batna 2 University**
       Department of Computer Science

Tutorial series 2

Exercise 0.1 *Determine the rank of the following matrices*

$$A = \begin{pmatrix} 2 & -1 & -3 \\ -2 & 3 & 7 \\ 8 & -4 & -12 \end{pmatrix}, B = \begin{pmatrix} 1 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 9 & 0 & 5 & 1 \end{pmatrix}, C = \begin{pmatrix} 2 & 2 & 3 \\ 1 & -2 & 0 \\ 5 & 5 & 15/2 \end{pmatrix}, and\ D = \begin{pmatrix} 3 & 4 & -5 \\ 1 & 4/3 & -5/3 \\ -4 & -16/3 & 20/3 \end{pmatrix}.$$

Exercise 0.2 *Find the solutions of the following systems*

$$\begin{aligned} x_1 + 3x_2 + 5x_3 + 6x_4 &= 1, \\ 4x_1 + x_2 - 2x_3 + 3x_4 &= -2, \\ x_1 - 3x_2 - 7x_3 + 8x_4 &= 3 \end{aligned} \qquad \begin{aligned} 2x_1 + 3x_2 + 4x_3 &= 3, \\ 2x_1 + 5x_2 + 4x_3 &= 5, \\ -2x_1 + x_2 - 7x_3 &= 1 \end{aligned}$$

$$\begin{aligned} 4x_1 + 2x_2 + 4x_3 + 7x_4 + 9x_5 + 6x_6 &= 1, \\ 5x_1 + 6x_2 + x_3 + 3x_5 + 7x_6 &= 3, \\ 4x_1 + 5x_2 + 2x_3 + 3x_4 + x_5 &= 1 \end{aligned}$$

Exercise 0.3 *Give the values of β if the rank of A is equal to 3.*

$$A = \begin{pmatrix} 5 & 6 & 1 \\ 10 & 2 & 4 \\ -5 & 3\beta - 2 & 3 \end{pmatrix}.$$

Exercise 0.4 *Let $AX = B$ be the linear system defined by*

$$\begin{pmatrix} 5 & 5 & 3 \\ 7 & 2 & \alpha/2 \\ 2 & 1 & 3/5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 14/5 \\ 2/5 \end{pmatrix}.$$

Give the values of α in the case where the given system has a unique solution and in the case where this system does not have solutions.

Exercise 0.5 *Let $AX = B$ be the linear system defined by*

$$\begin{pmatrix} 3 & 1 & 4 & 5 \\ 3 & 0 & 2 & 0 \\ 6 & 5 & 0 & 1 \\ 9 & 6 & 2 & -\alpha + 5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 7 \\ 7 \\ 0 \\ 5 \end{pmatrix}.$$

Find the values of α in the case where there exists a unique solution and in the case where there exist infinite solutions.