BATNA 2 University of Algeria.
Mathematics and Computer Science Faculty
Common Core in Mathematics and Computer Science Department

Probabilities and Statistics II.
Practical Exercises 1
Semester-3. L2 SCMI.

Exercise 1

Let X and Y be discrete random variables, with joint probability function given by

$$P_{XY} = \begin{cases} 1/2 & x = 3, y = 5 \\ 1/6 & x = 3, y = 9 \\ 1/6 & x = 6, y = 5 \\ 1/6 & x = 6, y = 9 \\ 0 & otherwise \end{cases}$$

1. Compute $E(X), E(Y)$ and $E(XY)$.
2. What do you conclude ?
3. Prove that if X and Y are independent then $E(XY) = E(X)E(Y)$.

Exercise 2

Compute C and $E(X)$ when the density function of X is given by each of the following.
(a)

$$f_X(x) = \begin{cases} C(x+1) & 6 \leq x \leq 8 \\ 0 & Otherwise \end{cases}$$

(b)

$$f_X(x) = \begin{cases} Cx^4 & -5 \leq x \leq -2 \\ 0 & Otherwise \end{cases}$$

(c)

$$f_X(x) = \begin{cases} \frac{1}{x^2} & x \geq 1 \\ \frac{C}{x^2} & x \leq -1 \\ 0 & Otherwise \end{cases}$$

Exercise 3

Prove the monotonicity and the linearity of the expectation of a discrete random variable

Exercise 4

(a). Compute the expectation of the following distributions:

1. Geometric distribution
2. Hyper-geometric distribution
3. Poisson distribution
4. Standard normal distribution $N(0, 1)$

(b) Using the properties of the expected value, conclude the expectation of the normal distribution $N(\mu, \sigma^2)$

Exercise 5

Suppose the joint probability function of X and Y is given by

$$P(X = x, Y = y) = \begin{cases} 1/7 & x = 5, y = 0 \\ 1/7 & x = 5, y = 3 \\ 1/7 & x = 5, y = 4 \\ 3/7 & x = 8, y = 0 \\ 1/7 & x = 8, y = 4 \\ 0 & otherwise \end{cases}$$

(a) Compute $E(X)$ and $E(Y)$.
(b) Compute $Cov(X, Y)$.
(c) Compute $Var(X)$ and $Var(Y)$.
(d) Compute $\rho(X, Y)$.

Exercise 6

Let X and Y have joint density

$$f(x,y) = \begin{cases} 2x^3y + 2y^3 & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & otherwise \end{cases}$$

Compute $\rho(X,Y)$.

Exercise 7

Suppose you roll two fair six sided dice. Let X be the number showing on the first die, and let Z be the sum of the two numbers showing.
(a) Compute$E(X)$.
(b) Compute $E(Z|X=1$.
(c) Compute $E(Z|X=6)$.
(d) Compute $E(X|Z=2)$.
(e) Compute $E(X|Z=4)$.
(f) Compute $E(X|Z=6)$.
(g) Compute $E(X|Z=7)$. .

Exercise 8

Let X and Y have joint density

$$f(x,y) = \begin{cases} 8xy & 0 \leq x, y \leq 1, \\ 0 & otherwise \end{cases}$$

(a) Compute $f(x)$.
(b) Compute $f(y)$.
(c) Compute $E(X|Y)$.
(d) Compute $E(Y|X)$.
(e) Compute $E(E(X|Y))$, and verify that it is equal to $E(X)$.