BATNA 2 University of Algeria.
Mathematics and Computer Science Faculty
Common Core in Mathematics and Computer Science Department

Probabilities and Statistics II.
Semester-3. L2 SCMI.
Practical Exercises II

Exercise 1

A rowing team consists of four rowers who weigh 152, 156, 160, and 164 pounds. Find all possible random samples with replacement of size two and compute the sample mean for each one.
Use them to find the probability distribution, the mean, and the standard deviation of the sample mean $\bar{X}$.

Exercise 2

Consider the following probability distribution.

X	0	1	5
$P(X = x)$	1/3	1/3	1/3

(a) Find $E(X)$.
(b) Find the sampling distribution of the sample mean for samples of size $n = 2$.

Exercise 3

(a) Let $X_1, X_2, ..., X_n$ be a sequence of Bernoulli random variables. Let $Y_n = \frac{X_1+X_2+\cdots+X_n}{n^2}$
Show that this sequence converges in probability to the zero random variable.
(b) Suppose Z_n follows Exponential(n) distribution and let $Z = 0$. Show that $Z_n \xrightarrow{P} Z$.

Exercise 4

Let $\{X_n\}$ and $\{Y_n\}$ for $n = 1, 2, ...$ be two sequences of random variables, defined on the sample space S . Suppose that we know that

$$X_n \stackrel{a.s}{\to} X, \quad Y_n \stackrel{a.s}{\to} Y.$$

Prove that $X_n + Y_n \stackrel{a.s}{\to} X + Y$.

Exercise 5

(a) Suppose that X_n converges to X in probability, Y_n converges to Y also in probability. Show that $X_n + Y_n$ converges to $X + Y$ in probability.
(b) Suppose that X_n converges to X in probability. Show that there exists a subsequence X_{nk} converging to X almost surely.
(c) Suppose that X_n converges in distribution to a deterministic constant c. Show that $X_n \to c$ in probability.

Exercise 6

Let $X_1, X_2, ...$ be a sequence of i.i.d. Uniform[0,1] random variables. We define the sequence Y_n as

$$Y_n = min(X_1, X_2, ..., Xn).$$

Prove the following convergence results independently (i.e, do not conclude the weaker convergence modes from the stronger ones).
(a) $Y_n \stackrel{D}{\to} 0$.
(b) $Y_n \stackrel{P}{\to} 0$.
(c) $Y_n \stackrel{a.s}{\to} 0$