Algebra tutorial series 1 (continued)

Exercise 1 *Consider the two vector spaces of* $\mathbb{R}^3$

$$E = \left\{(x, y, z) \in \mathbb{R}^3 / \qquad x + z = 0 \quad \text{and} \quad x + y - 2z = 0.\right\}$$

$$F = span\{u, v\}, \quad u = (1, -2, 1), v = (2, 0, 1).$$

1. *Determine a basis and the dimension of each of the subspaces :* $E, F, E \cap F$ *and* $E + F$.
2. *Do we have* $\mathbb{R}^3 = E \oplus F$?
3. *Verify that* $u = (2, -6, -2) \in E$ *and determine the coordinate of the vector* u *in the basis of* E.

Exercise 2 *Consider the following polynomials in* $\mathbb{R}_2[X]$ *:*

$$P_1 = X + 1, \quad P_2 = X^2 - 1, \quad P_3 = X^2 - 2X + 1, \quad P_4 = -X^2 + X + 6.$$

1. *Show that* $B = \{P_1, P_2, P_3\}$ *is a basis of* $\mathbb{R}_2[X]$.
2. *Determine the coordinates of* P_4 *in the base* B.

Exercise 3 *Let's consider* $E = \mathbb{R}^3$.
Prove that $F = \{(x, y, z) \in \mathbb{R}^3 / \qquad x - y + z = 0\}$ *and* $G = \{(x, x, x) \in \mathbb{R}^3\}$ *are complements in* E *(i.e.* $\mathbb{R}^3 = F \oplus G$*).*

Exercise 4 *Consider the vectors of* $\mathbb{R}^4$ *:* $u = (1, -2, 4, 1)$ *and* $v = (1, 0, 0, 2)$.

1. *Determine* $Vect(u, v)$.
2. *Complete the set* (u, v) *adding two vectors of the canonical basis of* $\mathbb{R}^4$ *in order to have a basis of* $\mathbb{R}^4$.
3. *In* $\mathbb{R}^3$, *let* $u = (2, 3, 5), v = (4, 6, 10)$ *and* $w = (-2, -3, -5)$ *. Find* $rg(u, v, w)$.

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