## Written exam

**I. a/**

1. $\forall x \in \mathbb{R}^*, \forall y \in \mathbb{R}^*, xy \neq 0. \cdots\cdots (T)$, since $x \neq 0$ and $y \neq 0 : (xy = 0 \Leftrightarrow x = 0$ or $y = 0)$.
2. $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, xy = 1 \cdots\cdots (F)$. Because if such an $x$ exists, for $y = 0, xy \neq 1$.

$\overline{2.}$ $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}/ \quad xy \neq 1$.

3. $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}/ \quad xy = 1 \cdots\cdots (F)$.

$\overline{3.}$ $\exists x = 0 \in \mathbb{R}/ \quad \forall y \in \mathbb{R}, \quad xy \neq 1$.

4. $\forall x \in \mathbb{R}, x + 1 \neq 0$ or $x + 2 \neq 0. \cdots\cdots (T)$. If $x + 1 = 0$ then $x = -1$ and so $x + 2 = 1 \neq 0$ and if $x + 2 = 0$ then $x = -2$ and so $x + 1 = -1 \neq 0$.

**b/** $f : E \to F$. $f^{-1}(F) = E$ and $f^{-1}(\emptyset) = \emptyset$.

**II.** $A = ]0, 6[\,, \quad B = \{x \in \mathbb{N}, \quad x < 3\}$. We have :

$$B = \{0, 1, 2\}\,, \qquad A \cap B = \{1, 2\}\,, \qquad A \cup B = [0, 6[\,, \qquad C_A^B \quad \textit{does not exist since } B \nsubseteq A,$$

$$A \cap \mathbb{N} = \{1, 2, 3, 4, 5\}\,, \qquad C_{\mathbb{R}}^A = ]-\infty, 0] \cup [6, +\infty[\,.$$