

Exercise 1

1. show that the mean square error of an estimator T equal to its variance plus its bias square :

$$MSE(T) = Var(T) + Bias(T)^2$$

2. Let X_1, X_2, \dots, X_n be iid samples from $X \rightsquigarrow Unif(0, \theta)$ (continuous). What is the moment estimator of θ ?
3. Let X_1, X_2, \dots, X_n be iid samples from $X \rightsquigarrow Poisson(\lambda)$. What is the maximum likelihood estimator of λ ?

Exercise 2

Compute the moment and maximum likelihood estimators of normal distribution $N(\mu, \sigma^2)$

Exercise 3

Prove the monotonicity and the linearity of the expectation of a discrete random variable
A random sample of 20 nominally measured 2mm diameter steel ball bearings is taken and the diameters are measured precisely. The measurements, in mm, are as follows:

2.02 1.94 2.09 1.95 1.98 2.00 2.03 2.04 2.08 2.07
1.99 1.96 1.99 1.95 1.99 1.99 2.03 2.05 2.01 2.03

Assuming that the diameters are normally distributed with unknown mean, μ , and unknown variance σ^2

- (a) find a 95% confidence interval for the mean.
- (b) find a 95% confidence interval for the variance.
- (c) find a confidence interval for the standard deviation.

Exercise 4

In a typical car, bell housings are bolted to crankcase castings by means of a series of 13 mm bolts. A random sample of 12 bolt-hole diameters is checked as part of a quality control process and found to have a variance of 0.0013 mm^2 .

- (a) Construct the 95% confidence interval for the variance of the holes.
- (b) Find the 95% confidence interval for the standard deviation of the holes