University year : 2023-2024
Department :MI
Module : Algebra 2

Tutorial Series(2)

Exercise 1 *Let F_1, F_2 be two subsets of $\mathbb{R}^3$ defined as :*

$$F_1 = \{ \ (x, y, z) \in \mathbb{R}^3 : \ x = y = z \ \}$$
$$F_2 = \{ \ (x, y, z) \in \mathbb{R}^3 : \ x + 2y = 3z \ \}$$

(1)- Show that F_1, F_2 are two vector sub-spaces of $\mathbb{R}^3$.
(2)- Find the basis of F_1, F_2 and give their dimensions $(dimF_1, \ dimF_2)$.
(3)- Is $\mathbb{R}^3 = F_1 \oplus F_2$?

Exercise 2 *Determine which of the following maps are **linear** .*

$$f : \mathbb{R}^2 \to \mathbb{R}^3, (x, \ y) \longmapsto (x + y, \ x - 2y, \ 0);$$
$$f : \mathbb{R}^2 \to \mathbb{R}^3, (x, \ y) \longmapsto (x + y, \ x - 2y, \ 1);$$
$$f : \mathbb{R}^2 \to \mathbb{R}^2, (x, y) \longmapsto (x - y, \ 2x + y);$$
$$f : \mathbb{R}^2 \to \mathbb{R}, \ (x, y) \longmapsto \ x^2 - y^2;$$

Exercise 3 *Let $f \in \mathcal{L}(\mathbb{R}^3, \mathbb{R}^2)$ be defined by :*

$$f(x, y, z) = (x + y + 2z, \ x - y)$$

(1)- Show that f is a linear application (linear map)
(2)- Determine $kerf$ and Imf.
(3)- Determine the basis of $kerf$ and Imf and deduce their dimensions $dimkerf$, $dimImf$. What can we conclude ?

Exercise 4 *Let $f \in \mathcal{L}(\mathbb{R}^2, \mathbb{R}^3)$ be defined by :*

$$f(x_1, x_2) = (x_1 + x_2, \ x_2, \ x_1)$$

(1)- Show that f is a linear application (linear map)
(2)- Determine $Rank(f)$. What can we conclude ?

Exercise 5 *We consider $B = \{e_1 = (1, 0, 0), e_2 = (0, 1, 0), e_3 = (0, 0, 1)\}$ the canonical base of $\mathbb{R}^3$ - Determine the expression of $f \in \mathcal{L}(\mathbb{R}^3, \mathbb{R}^4)$ such that :*

$$f(e_1) = (1, 2, -1, 2), \ f(e_2) = (2, 4, 4, -8), \ f(e_3) = (3, 6, 1, -2)$$