Course : Algebra 3
Chapter 4 : Vector spaces

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Tutorial series 4

Exercise 0.1 *Let* $<,>: \mathbb{R}^n \times \mathbb{R}^n \longrightarrow \mathbb{R}$ *be a function satisfying, for* $x, y \in \mathbb{R}^n$,

$$< x, y >= \sum_{i=1}^{n} x_i y_i. \tag{1}$$

Prove that $< x, y >$ *is an inner product on* $\mathbb{R}^n$.

Exercise 0.2 *Consider, for* $w \in C[a, b]$,

$$< f, g >= \int_a^b w(t) f(t) g(t) dt,$$

such that w *is a fixed positive function. Prove that* $< f, g >$ *is an inner product on* $C[a, b]$.

Exercise 0.3 *Consider*

$$< f, g >= \int_0^{\pi} f(t) g(t) dt. \tag{2}$$

Prove that $f(t)$ *and* $g(t)$ *are orthogonal with respect to (2) in* $C[0, \pi]$ *such that* $f(t) = \cos(t)$ *and* $g(t) = \sin(t)$.

Exercise 0.4 *Let*

$$< p, q >= \int_{-1}^{1} p(x) q(x) dx, \tag{3}$$

be an inner product on the vector space of polynomials of degree 2 or less, denoted by P_2. *Find an orthogonal basis for* P_2.

Exercise 0.5 *Prove that*

$$\{\frac{1}{\sqrt{2}}(v_1 - v_2), \frac{1}{\sqrt{2}}(v_1 + v_2)\}, \tag{4}$$

is an orthonormal set where $\{v_1, v_2\}$ *is an orthonormal set in an inner product space.*

Exercise 0.6 *Let* $x, y \in V$. *Prove that*

1. $| < x, y > | \preccurlyeq \| x \| \| y \|$, *(The Cauchy-Schwarz inequality).*

2. $\| x + y \| \preccurlyeq \| x \| + \| y \|$, *(The triangle inequality).*